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Exploring DFT-Based Techniques for Harmonic Phase Estimation, Transformation, and Synthesis

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Abstract- ***Many natural signals exhibit quasi-periodic behavior and can be effectively modeled as combinations of harmonic sinusoids, where the relative frequencies, magnitudes, and phases change over time. The waveform shapes of these signals provide valuable insights into the physical phenomena that generate them, making accurate estimation and modeling of these parameters crucial. While much attention has been given to frequency and magnitude estimation, phase estimation and modeling have received comparatively less focus in the literature. This paper addresses the accurate estimation of phase using Discrete Fourier Transform (DFT) techniques across six different scenarios involving two DFT-based filter banks and three distinct window functions. It demonstrates that phase estimation bias is less than 0.001 radians when the signal-to-noise ratio (SNR) is 2.5 dB or higher. By comparing the performance with the Cramér-Rao lower bound (CRLB), it is shown that one specific window function offers significant practical benefits by closely approximating the CRLB under favorable signal conditions while minimizing performance deviation under less favorable conditions. Additionally, this paper introduces a shift-invariant phase-related feature that captures the harmonic phase structure. This feature forms the basis for a new signal processing paradigm that simplifies the parametric modeling, transformation, and synthesis of harmonic signals, aiding in the understanding and reverse engineering of the phasegram. The theoretical framework and experimental results are presented in a reproducible manner, with code provided to facilitate the replication of results and support further research.***

Keywords: phase estimation, harmonic phase structure, harmonic magnitude and phase modeling, harmonic signal processing.

1. Introduction

1.1. Motivation: Many natural and synthetic quasi-periodic signals, including speech, singing, physiological signals such as ECG, music, and acoustic waves from mechanical system vibrations, have a harmonic structure of sinusoids whose magnitudes, phases, and underlying fundamental frequencies vary over time. Harmonic phases are crucial in defining the waveform shape of quasi-periodic signals and are therefore immensely informative about the physical phenomena that generate them. Examples include the periodic glottal excitation signal, which illuminates the physiological processes governing vocal fold vibrations in the larynx; and periodic acoustic signals from mechanical systems, which provide insights into whether these systems are operating correctly within predefined safety margins. Given that harmonic phases depend explicitly on time, they vary much faster than harmonic magnitudes and fundamental frequencies; this is challenging from the perspectives of signal analysis, estimation, interpretation, modeling, transformation, and synthesis [1–4]. Since accurate frequency and magnitude estimation of sinusoids have been extensively discussed in the literature [5–13], in this paper, we assume that these factors have been addressed and will instead focus on two problems related to phase. The first concerns the practical and accurate DFT-based phase estimation of individual sinusoids. This is instrumental in addressing a second problem: the parametric modeling of sinusoid phases within a harmonic structure in a way that is time-shift invariant, interpretable, insightful, and simplifies harmonic signal processing. To the best of our knowledge, this is the first time such a combined perspective is presented in a manner that is easily apprehensible and extensively illustrated. This paper demonstrates that these problems can be tackled practically, facilitated by Matlab code that replicates the main results and illustrations presented

1.2. Problem Statement: This paper focuses on practical methods for representing quasi-periodic signals through the concepts of structure and parametric modeling. By 'structure', we mean an identifiable form or organization, and by 'parametric modeling', we mean a simple mathematical formulation that captures and models the organization using a limited number of controllable parameters. We acknowledge in this paper that the signals discussed are quasi-periodic and locally stationary. Quasi-periodicity means that the waveform shape in a periodic signal varies slowly among at least three adjacent periods, even though the period length may vary. Local stationarity means that harmonic parameters, such as sinusoidal magnitude and frequency, vary slowly over time, allowing them to be considered approximately constant within a short, windowed region of the observed signal. In this paper, we focus on phase representation, estimation, and parametric modeling. We first address the concept of structure based on the three aspects that define a stationary periodic signal: (i) frequency, (ii) magnitude, and (iii) phase. Let us consider a real-valued signal x(t) that consists of L sinusoids, as follows:

$$x(t) = \sum_{\ell=0}^{L-1} A_\ell \sin(\Omega_\ell t + \phi_\ell) + s(t)\,,$$

$$\Omega_\ell = (\ell + 1)\Omega_0\,,$$

i.e., the frequency of each sinusoid is a multiple integer of a fundamental frequency represented by Ω0. Thus, this frequency organization represents a frequency-related feature that is intrinsic and, therefore, structural, to any periodic signal. This means that the only information in the frequency structure that is truly unique is the fundamental frequency. A second structural aspect that defines a periodic signal consists of the organization of the magnitudes of the different harmonics and can be given by the ratio between the magnitude of each harmonic (Aℓ) and that of the fundamental frequency magnitude (A0). If the latter is looked at as a gain, then a given periodic signal is characterized by a normalized magnitude-related feature vector, where the first value is one, and all other values (Aℓ/A0) help to define the waveform shape of the periodic signal. This magnitude related feature vector expresses the magnitude structure of a periodic signal, independently of time, and the fundamental frequency of the signal, provided that the waveform shape is locally preserved. A third aspect that contributes to defining the particular waveform shape of a periodic signal involves the relationships between the starting phases of the different harmonics, ϕℓ . Ideally, it would be interesting to characterize the waveform shape of a given periodic signal based on a normalized phase-related feature vector that, similar to the magnitude related feature vector, is independent of time and the fundamental frequency. This paper aims to show that such a time-shift invariant phase-related feature exists and that it can be extracted using fairly conventional spectrum analysis. Moreover, it can be modeled in simple and insightful ways. To this end, two problems need to be addressed. The first one involves estimating the harmonic starting phase values, ϕℓ , from a short segment of x(t) that is representative of the periodic signal. By 'representative', we mean that the short segment contains a few periods with a similar waveform shape, but its duration is not related to the period of the periodic signal. The second aspect involves establishing a model based on the estimated harmonic starting phases, which, when combined with harmonic magnitude information, helps to fully explain a given waveform shape in a way that is time-shift invariant and independent of Ω0. The next section further discusses these challenges from a practical perspective and motivates a paradigm in harmonic signal processing oriented toward the harmonic magnitude and phase structure, greatly facilitating signal modeling, transformation, and synthesis.

1.3. A Practical Approach to Harmonic Signal Processing The problem we address falls within the realm of spectrum estimation using Fourier analysis. Given our aim for practical signal processing that is suitable for real-time operation on low-cost platforms, we highlight simple technical approaches that utilize the discrete Fourier transform (DFT). This choice allows for the benefit of efficient realization

algorithms (e.g., the FFT [15]). In addition, we exclude non-causal or iterative processing in order to accommodate real-time operation on low-cost platforms. Thus, in a practical setting, simple spectrum estimation typically implies three operations: (1) sampling, (2) time–frequency (T-F) transformation, and (3) frequency, magnitude, as well as phase estimation: 1. A discrete-time version of the signal represented by Equation (1) is first obtained using a convenient sampling frequency (FS), i.e., we have the following:

$$X[k] = \sum_{n=0}^{N-1} x[n]w[n]e^{-j\frac{2\pi}{N}kn},\ k = 0, 1, \ldots, N-1\ ,$$

Finally, a suitable estimation procedure is used that takes the spectral coefficients, X[k], as input and delivers robust estimates of all the harmonic frequencies, magnitudes, and phases. These simple spectrum estimation steps enable the identification of important harmonic parameters such as the normalized fundamental frequency, ω0 = Ω0TS. On the other hand, if the parametric representation of harmonic magnitudes and phases is done in such a way that it does not depend on time or the fundamental frequency, it paves the way for a harmonic signal processing paradigm that promotes simplification, flexibility, and even insight in algorithms. In fact, in many applications, such as speech enhancement and time-scale or pitch-scale modification of speech [3], and special effects in singing and music [2], harmonic sinusoids Signals 2024, 5 844 are individually modified in terms of their frequency, magnitude, and phase trajectories, as suggested by Figure 1.

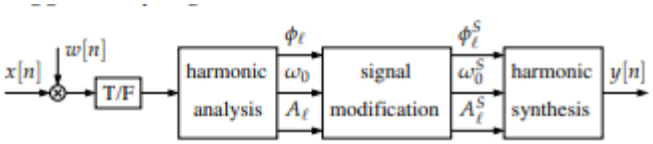

Figure 1. In many harmonic signal processing applications, individual harmonics are processed separately. Signal modification converts the original fundamental frequency (ω_0), the original harmonic magnitudes (A_ℓ), and phases (ϕ_ℓ), into new values (ω_0^S, A_ℓ^S, ϕ_ℓ^S) in the synthesis process.

However, this approach requires careful unwrapping of the phases of all harmonics such that their modifications are correct and do not suffer from the errors that may result from the wrapped phase representation in the interval [−π, π]. In addition, phase unwrapping, which is carried out for all harmonics on an individual basis and in a 'horizontal' manner, i.e., along the time axis, is itself prone to estimation errors. The most critical aspect of this approach, however, is that it is not insightful, i.e., it does not capture the overall, or holistic, harmonic phase structure, which means that, most likely, it does not explicitly control it. As a result, although phase coherence may be obtained on an individual sinusoidal basis, 'vertical' coherence may not be controlled and artifacts may result, the most common being known as 'phasiness' [16]. A more convenient paradigm in harmonic signal processing is represented by the block diagram illustrated in Figure 2.

According to this paradigm, harmonic magnitude and phase models are extracted that represent the holistic harmonic structure (or 'vertical' structure) in a way that is time-shift invariant, and independent of the fundamental frequency. This approach not only promotes insight into the harmonic signal structure but also greatly facilitates signal transformation. For example, the magnitude model (MM) and the phase model (PM) may change arbitrarily or may be interpolated in simple ways. Or, if the waveform shape is to be preserved, they may remain unchanged, and only the changes in the synthesis affect the fundamental frequency parameters (ωS 0 , A S 0 , ϕ S 0). In addition, the synthetic phase ϕ S 0 may be decoupled from the original phase ϕ0 since it can be easily synthesized using the value of ωS

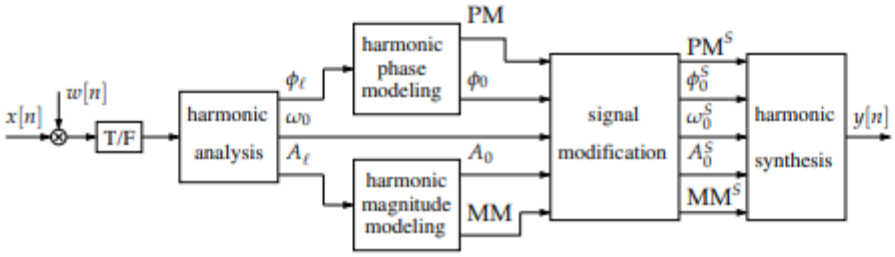

Figure 2. A convenient paradigm in harmonic signal processing: a holistic harmonic magnitude and phase structure represented by a shift-invariant and fundamental frequency-independent magnitude model (MM), and phase model (PM), respectively. Signal transformation just involves modifications to these models.

2. Robust DFT-Based Phase Estimation of Individual Sinusoids

This section focuses on the estimation of the initial phases (ϕℓ\phi_{\ell}ϕℓ) of individual sinusoids in a harmonic signal, as expressed in equation (1). The process begins after the signal undergoes simple uniform sampling, as indicated in equation (3), followed by multiplication by a window function w[n]w[n]w[n], leading to a transformation into the discrete-frequency Fourier domain, as shown in equation (4). As detailed in Section 1.1 and illustrated in Figures 1 and 2, we utilize a pre-existing harmonic analysis framework that not only provides an accurate estimate of the fundamental frequency (ω0\omega_0ω0) but also the harmonic magnitude estimates (AℓA_{\ell}Aℓ). In our simulations within this paper, we work with ground-truth test signals where these parameters are known in advance, which is equivalent to assuming error-free estimation of these parameters.

When examining the magnitude spectrum of a harmonic signal, it is tempting to estimate the starting phases of the harmonics by directly taking the phases of the DFT spectral lines (or DFT bins) that correspond to the local maxima in the magnitude spectrum. Specifically, if X[k]X[k]X[k] denotes the Fourier spectrum of the windowed harmonic signal, with k=0,1,…,N−K. However, this approach is incorrect because phase estimation requires considering the specific nature of the time-frequency transformation, the characteristics of the window function, and the relationship between each spectral peak in the magnitude spectrum and the fundamental frequency.

$$\omega_\ell = \Omega_\ell T_S = \frac{2\pi}{N}(k_\ell + \Delta_\ell) \,.$$

One of the key contributions of this paper is the demonstration that phase estimation does not necessarily depend on the accurate frequency estimation of individual sinusoids. As noted by Rife and Boorstyn, the most practical DFT-based frequency estimators use a two-step approach: first, a coarse search followed by a fine search. The coarse search is straightforward, involving peak picking, while the fine search step determines the accuracy of the frequency estimation. Typically, sinusoidal magnitude estimation is reliant on the fine search step in frequency estimation, meaning that the accuracy of magnitude estimation is tied to the precision of frequency estimation. However, this section establishes that phase estimation is not heavily influenced by the fine search step in frequency estimation. Instead, its accuracy is primarily determined by the signal-to-noise ratio (SNR) of the signal.

$$x[n] = A_\ell \sin(\omega_\ell n + \phi_\ell) + s[n] \,.$$

To demonstrate these principles, we consider six distinct cases, each resulting from the combination of two different DFT-based filter banks and three different window functions. For simplicity and to facilitate comparisons, we restrict our analysis to the examination of a single harmonic sinusoid, as

described in equation (1). After uniform sampling as outlined in equation (3), the signal is represented and processed accordingly.

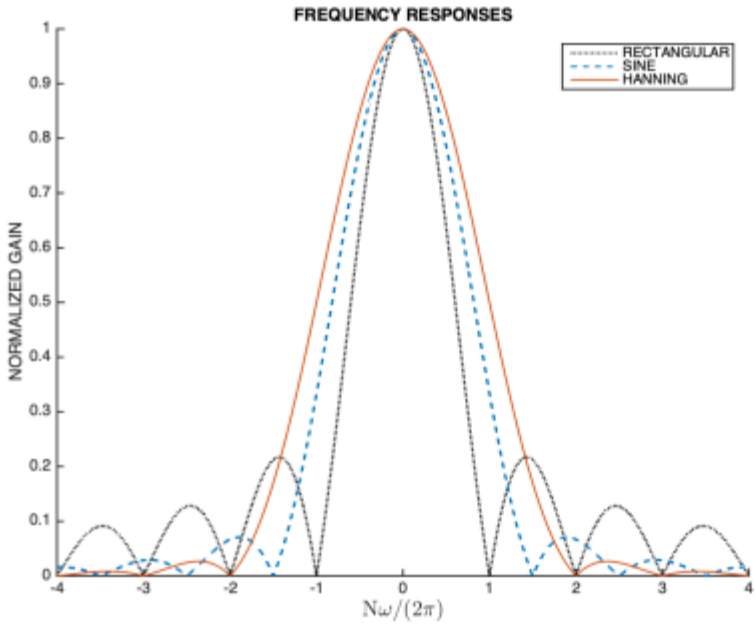

Figure 3. Normalized magnitude of the frequency response of the rectangular, sine, and Hanning windows in the range $-8\pi/N \leq \omega \leq 8\pi/N$. The frequency axis ($\omega$) is normalized by $2\pi/N$.

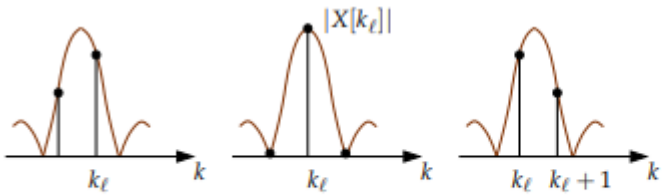

Figure 4. When $\Delta_\ell \in [-0.5, 0.5[$, the magnitude spectrum of the DFT of a rectangular-windowed sinusoid exhibits a local maximum for $k = k_\ell$. The plots illustrate the case when $\Delta_\ell = -0.5 + \epsilon$ (**left**), when $\Delta_\ell = 0.0$ (**center**), and when $\Delta_\ell = 0.5 - \epsilon$ (**right**), where $\epsilon < 0.5$ is a small real positive number.

Despite being usable, this result presents one practical difficulty since it requires that the fractional frequency ($\Delta\ell$) be estimated as accurately as possible [5,18]. This means that frequency estimation errors may propagate to the phase estimation. In order to avoid this, a more robust approach is available if, instead of estimating phase with respect to the origin of the time segment, estimation is performed with respect to the group delay of the DFT filter bank. Given that the window is symmetric, the group delay is constant and given by $\tau = (N - 1)/2$. Therefore, using this result and (6) and (9), we obtain the following:

It can be seen that the estimation error distribution is consistent with the relative magnitudes of adjacent spectral bins as illustrated in Figure 4. In fact, when $\Delta\ell$ approaches −0.5, or 0.5, the cumulative error increases relative to the case when $\Delta\ell = 0$, which is a consequence of the fact that the magnitudes of two adjacent spectral lines become comparable and significantly lower than the maximum value they can reach (when $\Delta\ell = 0$), which not only exacerbates leakage effects but also increases vulnerability to the noise influence. Due to the specific frequency response of the rectangular window, as Figures 3 and 4 highlight, then, when $\Delta\ell = 0$ and the SNR is infinity, the spectral leakage is zero, and the phase is estimated without error. The Matlab code generating Figure 5 is available (estimate PHASE_DFT_rect.m), which facilitates experimentation with other values of N, ℓ, or SNR.

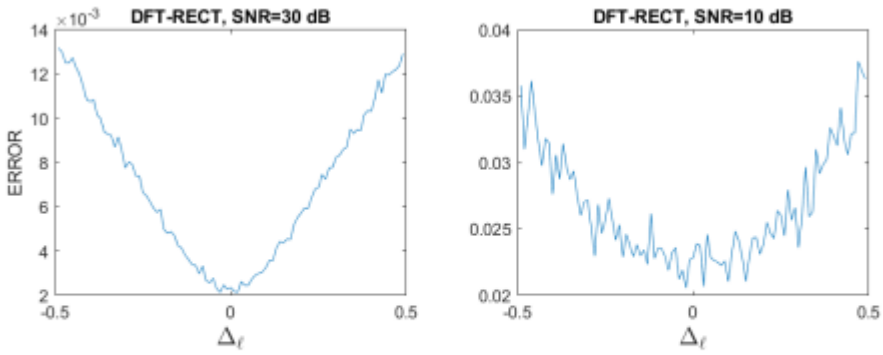

Figure 5. Cumulative phase estimation error as a function of the fractional frequency Δ_ℓ, when the DFT and the rectangular window are used. In the illustrated cases, $N = 128$, $\ell = 13$, SNR = 30 dB **(left)** and SNR = 10 dB **(right)**.

2.2. DFT and the Sine and Shifted Hanning Windows Other windows that provide better main-to-side lobe attenuation compared to the rectangular window are frequently used in spectrum estimation and FIR filter design [15]. Although standard Hamming or Hanning windows could be used in our analysis, we employ two related windows that are particularly significant in perfect reconstruction filter banks [19,20], such as those that are frequently used in audio coding and general analysis synthesis [21]. One window is known as the sine window and is defined as follows:

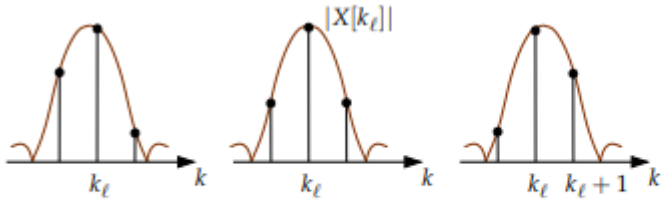

Figure 6. When $\Delta_\ell \in [-0.5, 0.5]$, the magnitude spectrum of the DFT of a sine-windowed sinusoid exhibits a local maximum for $k = k_\ell$. The plots illustrate the case when $\Delta_\ell = -0.5 + \epsilon$ **(left)**, when $\Delta_\ell = 0.0$ **(center)**, and when $\Delta_\ell = 0.5 - \epsilon$ **(right)**, where $\epsilon < 0.5$ is a small real positive number.

Although this figure resembles Figure 4, two important aspects are worth noting. First, the side lobe attenuation decay (not shown in these figures) is stronger for the sine window, when compared to the rectangular window [5]. Second, because the main lobe of the frequency response of the sine window is wider than the main lobe of the rectangular window, this means that when $\Delta\ell$ is such that the magnitudes of the spectral lines k = kℓ − 1 and k = kℓ become comparable, or the magnitudes of the spectral lines k = kℓ and k = kℓ +1 become comparable, these magnitudes are closer to the maximum value they can reach (which takes place for $\Delta\ell$ = 0.0) than what happens for the rectangular window. These concurrent reasons make it that relative to the case of the rectangular window, phase estimation using the sine window is likely to be more immune to the noise influence and to suffer fewer leakage effects. This can be confirmed by making a simple study on the phase estimation error, as it was described in the previous subsection for the rectangular window. Using the same simulation conditions, and the same phase estimation function (Equation (10)), we obtain the cumulative phase estimation error results that are illustrated in Figure 7. When compared to the results in Figure 5, it can be concluded that phase estimation appears to be more accurate when the sine window is used, especially when the SNR is high, which is a natural consequence of the smaller leakage caused by this window. Results are similar if the shifted Hanning window is considered instead. As we shall see in Section 2.5, more informative conclusions will emerge from a study of the phase estimation error variance

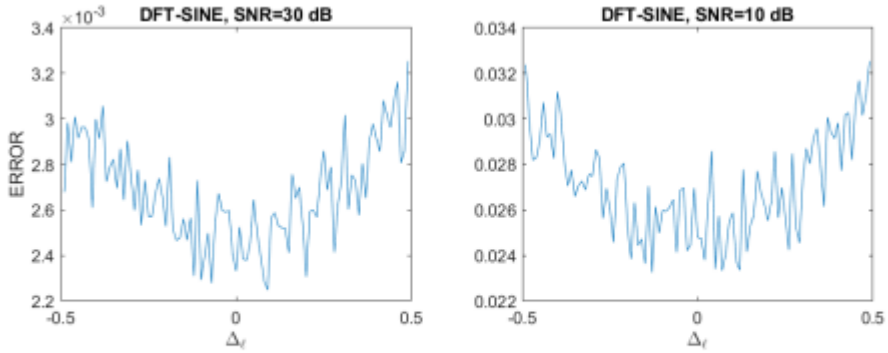

Figure 7. Cumulative phase estimation error as a function of the fractional frequency Δ_ℓ, when the DFT and the sine window are used. In the illustrated cases, $N = 128$, $\ell = 13$, SNR = 30 dB (**left**) and SNR = 10 dB (**right**).

Figure 8 displays representative results regarding bias. Results reveal that in all six cases, bias reduces as the SNR increases, as expected. However, there is not a specific combination of DFT-based filter bank and window that stands out. This remains true even after multiple runs of the simulations, although a relative degradation can be observed that persists for the tested rectangular window context (i.e., DFT filter bank and rectangular window) under more adverse Δ conditions (i.e., when Δ approaches −0.5 to 0.5), which is easily explained by the poor leakage characteristics of the rectangular window.

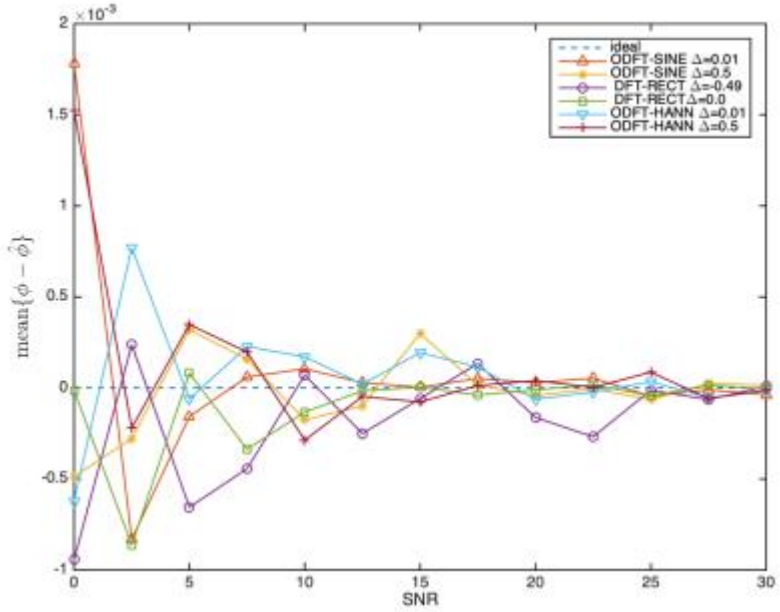

Figure 8. Mean over 100 Monte Carlo runs of the phase estimation error when N = 128, ℓ = 13, and when Δ takes on two extreme values depending on the estimator.

In general, it can be concluded that bias is fairly low, in the order or less than 0.001 radians for SNR equal to or larger than 2.5 dB. This represents less than 0.016% of the 2π dynamic range. The most significant results regard estimation error variance and are shown in Figure 9. It is an interesting and somewhat unexpected outcome that the rectangular window gives rise to the best results when the tested Δ conditions are more favorable (i.e., when Δ = 0.0), and to the worst results when the tested Δ conditions are more adverse (i.e., when Δ = −0.49). In the former case, the performance reaches the CRLB because in that ideal case, there is no leakage, as already noted at the end of Section 2.1, which means that the error variance is entirely due to noise contamination. It should be noted that in practice, this rarely happens with real-world, natural, signals as it is quite unlikely that the analyzed frequencies

are exactly aligned with the center frequencies of the sub-bands of the DFT-based filter bank. In the latter case, the performance shown by the same estimator is quite poor, which reveals that, in that case, leakage effects due to the rectangular window are quite strong, as the results in Figures 5 and 8 easily anticipate. In particular, the performance becomes asymptotic when the SNR exceeds 10 dB, which is commensurate with the known main-toside lobe attenuation of the rectangular window, in the order of 13 dB [15]. The performance of the sine and shifted Hanning windows fall in between the two extreme cases due to the rectangular window. In particular, for the same DFT-based filter bank and Δ conditions, the error variance performance of the sine window clearly exceeds that of the shifted Hanning window in the sense that a closer approximation to the CRLB is reached. This is more evident under the more favorable Δ test conditions (e.g., when Δ = 0.5) than under the more adverse Δ test conditions (e.g., when Δ = 0.01). A possible explanation may be linked to the relationship between the main lobe width of the magnitude of the frequency responses of those two windows, the relative prominence of the spectral coefficients inside that main lobe, the association with the discrete frequencies defining the different DFT channels (or sub-bands), and the near-end and far-end leakage characteristics of each window. The most impactful implication of these results is that under more general test conditions, the phase estimation error performance of the sine window is not only closer to the CRLB but also offers a lower deviation when signal conditions are more adverse. For these reasons, it can be considered that the performance of the sine window is better behaved and, thus, it will be used in the remainder of this paper.

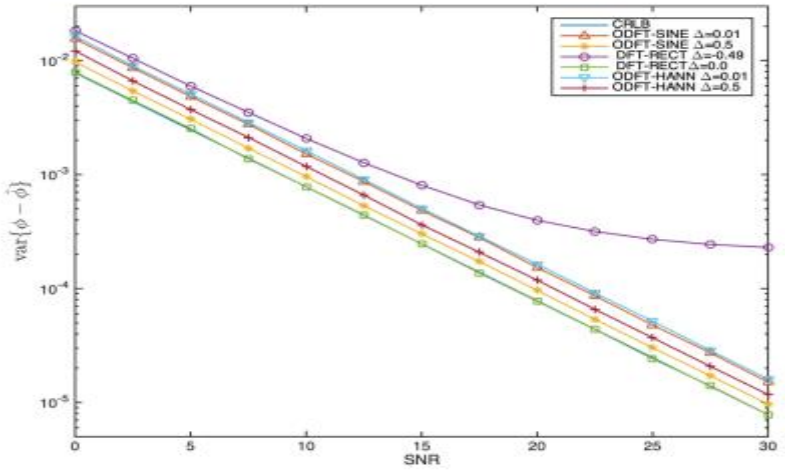

Figure 9. Variance of the phase estimation error when N = 128, ℓ = 13, and when Δ takes on two extreme values depending on the estimator. The Cramér–Rao lower bound is also represented although it is not too visible since it is overlapped by the DFT-RECT results when Δ = 0.0.

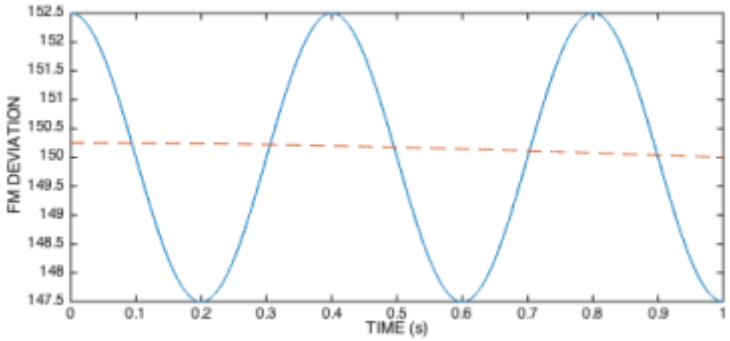

Figure 10. Two FM deviation cases characterizing test signals: FM = 2.5 Hz (solid line) and FM = 0.25 Hz (dashed line).

The sawtooth waveform is illustrated in Figure 11 for two noise contamination scenarios that we consider in our simulations: when it is mild (SNR = 30 dB), and when it is strong (SNR = 10 dB).

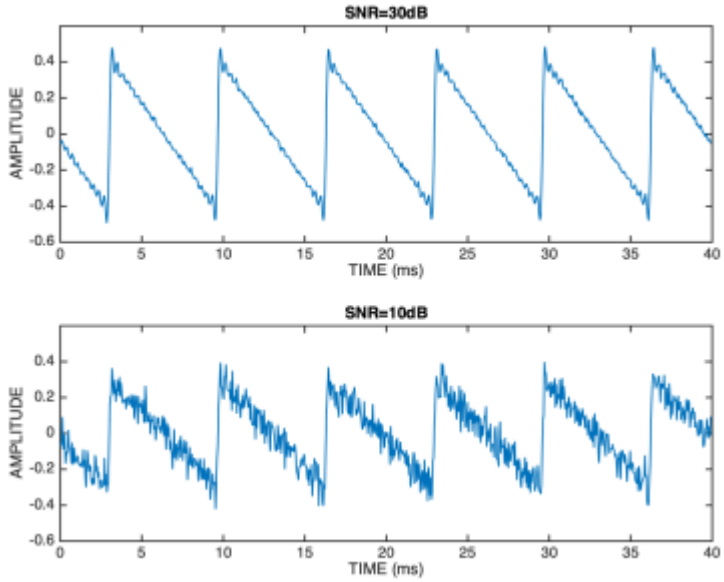

Figure 11. Illustration of the influence of noise on the sawtooth test signal when SNR = 30 dB (top), and when SNR = 10 dB (bottom).

DFT-based non-parametric spectrum estimation techniques are commonly used to analyze, represent, and interpret the spectral properties of a given signal for which no strong assumptions are made. Given that those spectral properties can be characterized in terms of magnitude and phase, typically, 2D plots and 3D plots can be used in each case, although only seldom in the case of phase representation. In the case of the magnitude spectrum, also known as a periodogram, a 2D plot is simply obtained by taking the absolute value of the short-time DFT of a windowed region of the signal as a function of frequency. This is illustrated, for example, in Figure 15. The graphical representation can be linear or logarithmic, either on the horizontal or vertical axis, or on both axes. When the goal is to observe how the magnitude spectrum evolves through time, a 3D representation is created by abutting several magnitude spectra, next to each other, and Signals 2024, 5 865 where a colormap is used to represent power spectral density (PSD). A new magnitude spectrum is obtained by sliding the short-time DFT window over the signal by a certain hop size that, typically, is less than the window length. Such a 3D representation is known as a spectrogram. Usually, the horizontal axis represents time, the vertical axis represents frequency, and the third dimension, which is perpendicular to the time–frequency plane, is represented by a specific color of a colormap denoting a suitable PSD range. A simple example is illustrated in Figure 26a that corresponds to the first four harmonics of a sawtooth signal and whose fundamental frequency is F0 = 187.34 Hz.

4. Conclusions: This paper focused on a harmonic signal analysis, modeling, and processing paradigm that eases significantly the representation, transformation, and synthesis of harmonic signals, especially from the point of view of the phase information. In the first part of the paper, practical DFT-based approaches that build on a filterbank perspective were discussed for estimating the starting phases of individual sinusoids. Their performance was characterized by considering the CRLB for the variance of an unbiased phase estimator. In particular, it was shown that contrary to harmonic frequency and magnitude estimation, accurate phase estimation depends only on ‘coarse search’ and not on ‘fine search’, which makes the estimation more robust. Six phase estimation alternatives were studied by combining two DFT-based filter banks and three different window functions. Results were explained in a reproducible manner. In the second part of the paper, it was shown that the starting phases of individual sinusoids that are harmonically related may be converted into a phase-related feature (NRD) that

expresses the holistic phase structure of a harmonic signal, has the advantage of being time-shift-invariant, helps to explain the waveform shape of a quasi-periodic signal, and helps to provide insight into the physical process that generates it. Finally, it was shown that the unique information that exists in a phasegram resulting from a stationary harmonic signal consists of the starting phase of the fundamental frequency and the NRD feature vector. Matlab code (https://github.com/Anibal-Ferreira/demo_AccPhaseEst, accessed on 1 January 2024) is provided that illustrates the most relevant concepts and results that are discussed in the paper. Many application scenarios may benefit from the results in this paper paving the way for new research results, namely speech coding, pitch, and time-scale modification of speech, singing, audio, and music; speech enhancement; whispered-speech to voice speech conversion and voice rehabilitation; audio forensics; physiological signal analysis and diagnosis (e.g., using ECG signals); and monitoring of the operation of mechanical systems using sound.

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